Bank Deposits and Credit in General Equilibrium*

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Abstract

We present a general equilibrium model of money with bank deposits and credit. Banks have two roles: first they act as safe-keepers of agents' values, second they act as transaction operators because they are able to identify agents. We show that there exists an equilibrium where money co-exists with bank deposits although interest rates payed on deposits are positive. Further, we compare our model to the basic framework where banks only act as safe-keepers and are not allowed to issue loans. Finally, we show that loans are welfare improving and that the money multiplier is decreasing over the inflation rate.

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1 Introduction

Apart from their traditional role as a safe-keepers of values, banks' primary activities include borrowing and lending money. We present a general equilibrium model with such traditional banks, i.e. a model where bank deposits, cash and credit co-exist.

There is an endogenous risk of theft in our economy. Hence, holding liquidity in the form of cash may be costly. In other words, agents with cash in their pockets may bump into a thief who tries to steal their money. As a result there is a (historic) role for banks to act as safehouses for values. Therefore, honest agents have an incentive to deposit a measure of their liquidity holdings on a bank account although they have to pay a deposit fee. He, Huang, and Wright (2005, 2008) study such a framework in a general equilibrium model with indivisible and divisible money. Our model extends their framework by introducing a credit market where agents may get a bank loan which is financed by unused bank deposits. We then show that there is an equilibrium with cash, credit, bank deposits and endogenous crime. Moreover, we show that there is even an equilibrium where cash, credit, and bank deposits co-exist when the endogenous crime rate went down to zero in equilibrium (agents remain indifferent between becoming thieves or remaining honest). This is an interesting result because for certain inflation rates, competitive banks are willing to pay a positive interest rate on deposits, and still agents will not deposit all of their money on the bank account. The reason is that there is a trade-off between the endogenous crime rate and the measure of bank deposits. The more money agents put on their bank accounts, the less crime there will be. However, for certain inflation rates the interest rate payed on the deposits is too low compared to the risk of theft. So, an agent will not want to deposit everything on his bank account. There is another aspect of bank deposits here. Bank deposits do not only offer an agent a safe place for their liquidity, but, as this is a general equilibrium model, they also offer the opportunity for future loans. This can be seen as a kind of insurance against the "risk" of becoming an agent in need of cash, i.e. a buyer.

We present and compare two versions of this model. The first one is the version of He, Huang, and Wright (2008) (but adapted to our timing) where the banks' only role is that of a safekeeper. In the second version, we expand the model by allowing banks to use their deposits for issuing loans and hence give them also the role of a transaction operator. Our model is in discrete time where a continuum of agents lives forever. Each period is divided into three sub-periods. First, agents enter a centralized Walrasian market where they trade goods for cash. Further in the case with bank loans, they use this market to pay back all outstanding interest rates and settle any other claims. After the centralized market closes, preference shocks are realized and some agents meet a thief. Then, a credit market opens for sellers and buyers, where they may readjust their deposits and get bank loans which have to be payed back in the consecutive CM. Finally, a decentralized goods market opens where honest agents meet pairwise to trade special goods for money.

For the first, basic version without bank loans, we present corrected results for the equilibria presented in He, Huang, and Wright (2008). (Our different timing is not relevant in this case.) The main big difference to their results is that the expected cost of holding cash is non-constant in any case, i.e. for any interest rate. This stems from the fact that we take into account the marginal effect the deposit fees have on a buyer's cash constraint. We can show that the expected cost of holding cash increases with an increase in the interest rates before falling towards zero after banks shut down. One explanation might be the structure of the fees which have to be payed per unit of deposit. Thus, an increase in the interest rates increases the total cost per unit of bank deposit inducing agent's to put less cash on their bank accounts, which in return increases the expected cost of holding cash.

In our extended model of the second version, we allow private banks to use the deposits that are not withdrawn by any agents during the credit market (this includes deposits from sellers for example) for issuing loans to agents in need of liquidity, i.e. the buyers. As we assume full enforcement of repayment of loans, these loans basically consist of the deposits from honest agents who met a thief and those who did not have a preference shock of becoming a buyer in the consecutive decentralized market. Repayment of loans takes place in the consecutive centralized market at a market interest rate. As in the general case without bank loans, a too high inflation rate makes it very costly to become a thief because of his exposure to the inflation tax. Hence, the equilibrium crime rate goes down to zero at some critical level of inflation. The same is true for banks. They shut down when the inflation rate rises above a critical level. There are two reasons for this result. First, above this critical level, there is no crime anymore and second agents will not raise enough money in the centralized market (due to the high inflation tax) that would justify the benefit from getting loans, i.e. the insurance component of banking. Hence, we show that the same critical equilibrium interest rate results as in the basic version of our model, i.e. above this rate, neither thieves nor banks operate. On the lower end of the interest rate, we show that the lowest possible equilibrium interest rate is the same as in the case without loans. Only above this potentially negative interest rate, a monetary equilibrium exists. Between these extremes, the interesting things we discussed above happen.

Note that we only study monetary equilibria. However, in future versions of this paper we might want to extend our discussions to study equilibria, where there only exist banks. Let me refer to the Conclusion on page 31 for further discussions on this interesting issue.

Finally, comparing the two versions, we show that loans have a beneficial effect as they allow cash-constrained agents to consume more. Further, it is also possible to draw the multiplying effect of the credit market. We show that the multiplier decreases over the inflation rate towards one.

Literature We model our banks as in He, Huang, and Wright (2005, 2008). These two papers model the role of banks as historic safe-keepers. Their basic assumption is that holding large amounts of cash is not entirely costless because it may be stolen. On the other hand, putting money on a debit account is safe and comfortable but only available at a fee. The paper published first analyzes their model with indivisible money whereas the later paper analyzes it in the more general framework of Lagos and Wright (2005) with divisible money. Probably the first paper that introduced banks into a microfounded model of money is Berentsen, Camera, and Waller (2007). Their approach to create a role for banks is to regard them as transaction operator with a unique technology that allows identification of otherwise anonymous agents. Hence, in contrast to "normal" agents who will never give a private credit or an IOU to their trading partners, the bank in Berentsen, Camera, and Waller (2007) can identify each agent and therefore also enforce repayment (or force exclusion from the financial market). So, our model presents a combination of the two types of banks discussed above. On the one hand, our banks here are pure safe-keepers, on the other hand they are transaction machines between agents with low and high liquidity needs. Berentsen and Monnet (2009) analyze optimal monetary policy in a channel system using the framework of Lagos and Wright (2005) with competitive decentralized markets as in Rocheteau and Wright (2005). Between the centralized settlement market and the decentralized goods market they introduce a money market where agents can trade credits to adjust for their expected liquidity needs. This credit market is what we also introduce here in our paper. It allows agents to do all the banking business before the banks shut down in the decentralized market.

The paper continuous as follows. Section 2 presents the environment. In Section 3, we discuss our model. The equilibria for the two versions of our model are then presented in Section 4 and welfare is analyzed in Section 5. We conclude in Section 6.

2 Environment

We construct a dynamic general equilibrium model with a [0,1] continuum of agents who live forever. Time is discrete and we assume that each period t is divided into three sub-periods, namely a centralized market CM, a credit market, and a decentralized market DM. Every period begins with a centralized market where all agents consume and work. The credit market allows agents to readjust their money holdings and to prepare for the final sub-period, the decentralized market where agents meet pairwise and trade a specialized good for money. Figure 1 presents the timing of this model:

Figure 1: Model

The CM is a frictionless Walrasian market with utility $U(x_t) - h_t$, where x_t is consumption, h_t is labor (e.g. hours worked), and $U'(x_t) > 0 \geq U''(x_t)$. The other goods market, the DM, is a highly decentralized market characterized by random, anonymous, bilateral matching. Because of a standard double coincidence problem, trade is difficult in the DM. We assume that with probability σ, agent i wants to buy the specialized good agent j produces. With probability σ, agent j wants what agent i produces and with probability $(1-2\sigma)$ neither wants what the other produces. For simplicity let us assume a lack of double coincidence of wants, so that we can ignore pure barter (this is easily relaxed). As they are non-storable, all goods are produced for immediate trade and consumption, as for example service goods. Consuming a specialized good conveys utility $u(q_t)$ where we assume $u'(q_t) > 0 > u''(q_t)$, $u(0) = 0$, and $u'(0) = \infty$. The producer of the specialized good suffers from disutility $c(q_t)$ with $c'(q_t) > 0$, $c''(q_t) \geq 0$, and $c(0) = 0$. For future reference, let q^* be the solution to $u'(q_t) = c'(q_t)$. In contrast to the standard model of Lagos and Wright (2005), agents learn about their preferences (being a buyer or a seller) at the end of the CM without loss of generality but with the advantage that they know their role already when entering the credit market.

As it is a standard approach in the literature of monetary theory, we assume some form of *anonymity* to prevent agents to offer a personal IOU or private credit in exchange for output, promising to pay in the next centralized market where they can acquire the funds by supplying labor for example. This form of anonymity makes a tangible medium of exchange essential. This medium of exchange is intrinsically useless, and we call it *money.* To introduce monetary policy, let M_t be the aggregate stock of money at the start of period t. We assume that it evolves over time according to $M_{t+1} = (1 + \pi_{t+1})\, M_t$ for all periods t. In steady state, π_{t+1} is the inflation rate. The budget constraint of the government is

$$\pi_{t+1}\, M_t - T_t = p_t\, G_t$$

where T_t is a nominal lump sum transfer, G_t is exogenous real government consumption, and p_t is the price level in period t. We do not want to discuss the details of how government sets policy, as this is not crucial for our analysis. Therefore, let us assume that the government maintains a balanced budget over time, i.e. $G_t = 0$ for all t.

Let us pin down the nominal interest rate by the Fisher equation

$$1 + i_t = \frac{1 + \pi_{t+1}}{\beta} \tag{1}$$

where β is the discount factor between one DM and the next CM with $0 < \beta < 1$. This definition allows us to interchangeably set M_t, π_{t+1}, or i_t. Note that inflation is a tax on money holdings, regardless of who holds it, i.e. it is a tax on activities that use money.

Agents can be of two types in our framework. *Honest agents* trade goods in all markets (except of course the credit market which is purely for resettlement). *Thieves* try to steal money from all agents they meet. Note that we analyse only equilibria where agents are indifferent between being honest or dishonest. Possible switching between types takes place only at the beginning of each period in the centralized market.

The existence of these two types of agents promotes a role for a safe place where honest agents can deposit some or all of their money in order to reduce the risk of loosing everything to a thief. Historically, banks have taken these roles as safe places for valuables. Hence, let us assume that there exist a continuum of homogeneous private *banks.* These banks have a particular technology that allows them to identify agents. Each agent has the possibility to use a perfectly safe bank account as in He, Huang, and Wright (2005, 2008) to deposit a measure of his money holdings. In the credit market, agents can resettle their bank deposits and money holdings, and buyers in particular will withdraw the necessary amount for the consecutive decentralized market. The reason for that is that we assume similarly to Berentsen, Camera, and Waller (2007) that banks do not operate in the DM and can therefore not distinguish agents in this market. Hence, it is not possible to use bank deposits for buying goods in the DM. This is in contrast to the model of He, Huang, and Wright (2008) without loans, where bank deposits are equivalent to cash. Let us assume that there is free entry in the banking sector. Second, there are no incentive problems, like absconding with the funds, since banks are assumed to operate in the frictionless centralized markets where funds can be raised easily. To make it clear, private banks here are depositories that hold an agent's cash and give him a safe claim that can be exchanged back to cash in the credit market. Further, they are allowed to issue loans that have to be fully backed with their current deposits. This is similar to the banks in Berentsen, Camera, and Waller (2007), although these banks act more like transaction machines between agents with different liquidity needs because deposit only takes place in the credit market and buyers have no incentive to deposit any money in the centralized market. Depositing money on a bank entails a fee, $\phi \in \mathbb{R}$. To model this, we assume banks have an exogenous resource cost $a > 0$ per dollar on deposit, in terms of the CM good. Let us assume for the sake of simplicity, that agents who withdraw their deposits in the credit market (i.e. buyers) have to pay the operating cost a to the bank right away. All other agents pay the endogenous fee ϕ. In order to analyze both the cases with and without loans, let us assume that banks are required to keep a fraction ρ of deposits on reserve, while in principle the rest can be out on loan. If $\rho = 1$, banks do not issue any loans, otherwise with $\rho = 0$ they may issue all their unused deposits as loans. There is no inside money produced by the banks as we assume that only deposited money can be lend to agents. The interest rate to be payed on bank loans, l, in the consecutive centralized market is r^l.

Whereas bank deposits are perfectly safe, holding cash is not entirely risk-free. If an honest agent meets a thief with probability λ, there is a chance (with probability γ) that all his cash is stolen. The success parameter γ is exogenous whereas λ is completely endogenous and defined by the indifference condition between being honest or dishonest. Let us further assume for simplicity that goods are not transferable and cannot be stolen.

3 Model

Every period starts with a CM where generalized goods are traded and money may be deposited on a bank account. After this market closes, some agents meet a thief in the consecutive sub-period and preference shocks are realized. In a credit market, honest agents who did not meet a thief may readjust their deposits and get bank loans which have to be redeemed in the consecutive CM. Finally, a decentralized goods market opens where buyers and sellers meet pairwise to trade special goods for money.

A measure $\lambda \in (0, 1)$ of the population behaves as thieves in the credit market. These thieves try to steal honest agents' money holdings and succeed with probability $\gamma \in (0, 1)$. Hence, the risk of carrying cash is $\lambda \gamma$. Thieves are indistinguishable from all other agents in the centralized markets and do not participate in any "honest" activity (like trading goods, doing banking business, etc.) in the credit market and the DM. We could introduce consumption or punishment in the DM which would change the details but would not alter the main insights. To minimize the risk of meeting a thief, let us assume the existence of *banks* that can perfectly identify each agent in the CM and the credit market, and thus keep the deposits safe from theft. Agents need cash to buy goods in the DM but in the CM and the credit market, the only difference between cash and bank deposits is safety. In the CM, agents may choose to bring a fraction b of their total liquidity holdings to the bank. On all amounts deposited, the bank charges a fee $\phi \in \mathbb{R}$, where we assume that this fee is charged before the DM opens. Buyers who withdraw their money from the bank account pay the operating cost a to the bank right away.

After the CM closes, preference shocks are realized and agents meet thieves. Sellers and buyers get a last chance in the credit market to readjust their liquidity holdings. Buyers may take out loans to be redeemed in the consecutive CM at an interest rate r^l whereas all other agents (who did not meet a thief) may deposit the remaining cash on their bank account.[1]

Let us analyze a steady state version of this model. Hence, we drop the time subscripts. Within a period, variables with hats, $\hat{}$, stand for variables from the consecutive sub-period.

The value functions for agents entering the CM with money holdings m and loans l are denoted by $W^t(m)$ and $W^h(m, l)$ for thieves and honest agents, respectively. Let us denote the value functions for entering the credit market as $\Gamma^i(m, b)$ where $i \in \{t, b, s, n\}$ stands for thieves, buyers, sellers, or agents who did not match up. The Bellman equations in the DM are denoted as $V^i(m, l)$ for all $i \in \{b, s, n\}$.

[1]Although bank loans are accessible to all honest agents, it is obvious that in equilibrium only buyers ask for a bank loan.

The Bellman equation for a thief in the centralized market is:

$$\begin{aligned} W^{t}(m) &= \max_{x,h,\hat{m}} \left\{ U(x) - h + \Gamma^{t}(\hat{m}) \right\} \\ \text{s.t} \quad x &= h + \frac{m + T - \hat{m} + D}{p} \end{aligned} \tag{2}$$

where $\hat{m}$ is the money taken out of the CM and D the profit from owning (shares in) banks, with $D = 0$ in an equilibrium with free entry. Let us consider monetary equilibria, where the price p is strictly smaller than infinity at every date t. Because of the setup of our model, thieves can never meet a seller to buy goods. Let us further assume, that they are not allowed to borrow or deposit money from the bank. Hence, they will not need any money in the decentralized market, i.e. $\hat{m} = 0$, $x^{*} = U'^{-1}(1)$, $h^{*} = x^{*} - (m + T + D)/p$.

An honest agent enters the CM with money holdings m and bank loans l. He then solves the following problem:

$$\begin{aligned} W^{h}(m,l) = \max_{x,h,\hat{m},b} \Big\{ & U(x) - h + \lambda\,\Gamma^{h,t}(\hat{m},b) + (1-\lambda)\,\sigma\,\Gamma^{b}(\hat{m},b) \\ & + (1-\lambda)\,\sigma\,\Gamma^{s}(\hat{m},b) + (1-\lambda)\,(1-2\sigma)\,\Gamma^{n}(\hat{m},b) \Big\} \\ \text{s.t} \quad x = {} & h + \frac{m + T - \left(1 + r^{l}\right) l - \hat{m} + D}{p} \end{aligned} \tag{3}$$

An honest agent maximizes his expected utility over the consumption of the generalized good x, the quantity to produce in the CM, h, the amount of money to bring into the next sub-period, $\hat{m}$, and the measure of liquidity holdings to bring to the bank, b. His expenditures have to be equal to the amount of money from the last period, m, plus the lump-sum transfer T, minus the amount of money for the next sub-period, $\hat{m}$, and minus the loan to be payed back to the bank (including interest rates), l. The profit from owning banks is $D = 0$ in an equilibrium with free entry.

The first-order conditions for honest agents are

$$U'(x) - 1 = 0 \tag{4a}$$

$$\begin{aligned} & -\frac{1}{p} + \lambda\,\frac{\partial\,\Gamma^{h,t}(\hat{m},b)}{\partial\,\hat{m}} + (1-\lambda)\,\sigma\,\frac{\partial\,\Gamma^{b}(\hat{m},b)}{\partial\,\hat{m}} \\ & + (1-\lambda)\,\sigma\,\frac{\partial\,\Gamma^{s}(\hat{m},b)}{\partial\,\hat{m}} + (1-\lambda)\,(1-2\sigma)\,\frac{\partial\,\Gamma^{n}(\hat{m},b)}{\partial\,\hat{m}} = 0 \end{aligned} \tag{4b}$$

$$\begin{aligned} & \lambda\,\frac{\partial\,\Gamma^{h,t}(\hat{m},b)}{\partial\,b} + (1-\lambda)\,\sigma\,\frac{\partial\,\Gamma^{b}(\hat{m},b)}{\partial\,b} \\ & + (1-\lambda)\,\sigma\,\frac{\partial\,\Gamma^{s}(\hat{m},b)}{\partial\,b} + (1-\lambda)\,(1-2\sigma)\,\frac{\partial\,\Gamma^{n}(\hat{m},b)}{\partial\,b} = 0 \end{aligned} \tag{4c}$$

$$x - h - \frac{m + T - \left(1 + r^{l}\right) l - \hat{m}}{p} = 0 \tag{4d}$$

with the envelope conditions

$$\frac{\partial W^h(m,l)}{\partial m} = \frac{1}{p} \tag{5a}$$

$$\frac{\partial W^h(m,l)}{\partial l} = -\left(1+r^l\right)\frac{1}{p} \tag{5b}$$

Note that the amount of money taken into the next subperiod does not depend on the amount currently held, i.e. m_t is independent of $\hat{m}_t$ for all periods t, as in the basic framework of Lagos and Wright (2005). The reason for this is the quasi-linear utility agents have in the SM which helps to keep the distribution of money holdings tractable.[2]

All agents who did not get in contact with a thief will eventually use the credit market to readjust their liquidity holdings. All non-buying agents may deposit the remaining cash on the bank account at the same conditions (bank deposit fee ϕ). Let us assume without loss of generality that these agents do not want to reduce their deposit holdings (as the fee on the already deposited money is charged anyway and holding cash has no advantage). Buyers may ask the bank for a loan to be payed back in the consecutive CM and withdraw the deposits needed in the DM (remember that only money can be used in the DM to buy goods). The Bellman equations in the credit market are as follows.

Buyers solve the following problem in the credit market:

$$\Gamma^b(m,b) = \max_{l<\bar{l}} V^b\left[m\,(1-b\,a)+l,l\right] \tag{6}$$

where l is the loan from the bank and $\bar{l}$ is a borrowing constraint. The money holdings they bring into the DM, $\hat{m}$, consists of the money from the CM minus the bank fee plus the loan.

The first-order conditions are

$$\frac{\partial \Gamma^b(m,b)}{\partial l} = \frac{\partial V^b(\hat{m},l)}{\partial \hat{m}} + \frac{\partial V^b(\hat{m},l)}{\partial l} - \mu^b = 0 \tag{7a}$$

$$\mu^b\left(l-\bar{l}\right) = 0 \tag{7b}$$

where $\mu^b \geq 0$ is the Lagrange multiplier for the borrowing constraint.

The Bellman equations in the credit market for sellers and non-matched agents are:

$$\Gamma^s(m,b) = \max_{b\leq\hat{b}\leq 1} V^s\left[m\left(1-\hat{b}\,\phi\right),\cdot\right] \tag{8}$$

$$\Gamma^n(m,b) = \max_{b\leq\hat{b}\leq 1} V^n\left[m\left(1-\hat{b}\,\phi\right),\cdot\right] \tag{9}$$

[2] See for example Faig (2008), Rocheteau, Rupert, Shell, and Wright (2008), or Clausen and Strub (2009).

In fact, what these agents decide is whether they want to increase the measure of cash deposited on their bank account, $\hat{b}$. The first-order conditions are

$$\frac{\partial \Gamma^j(m,b)}{\partial \hat{b}} = -m\,\phi \frac{\partial V^j\left[m\left(1-\hat{b}\,\phi\right),\cdot\right]}{\partial \hat{m}} + \mu^{j1} - \mu^{j2} = 0 \tag{10a}$$

$$\mu^{j1}\left(-\hat{b}+b\right) = 0 \tag{10b}$$

$$\mu^{j2}\left(\hat{b}-1\right) = 0 \tag{10c}$$

As we noted earlier on page 8, a thief will – in equilibrium – bring no money into the credit market. Hence, his values for the credit market are

$$\Gamma^t(0) = \lambda\,\beta\,W^t_{+1}(0) + (1-\lambda)\left\{\gamma\,\beta\,W^t_{+1}\left[\bar{m}\left(1-\bar{b}\right)\right] + (1-\gamma)\,\beta\,W^t_{+1}(0)\right\} \tag{11}$$

taking λ, the money holdings of the honest agent $\bar{m}$, and the measure of his bank deposits, $\bar{b}$, as given. With probability λ, a thief meets another thief, where nothing is to steal. With probability $(1-\lambda)$, a thief meets an honest agent, succeeds with his theft with probability γ, and fails with probability $(1-\gamma)$. In case he is successful, he will enter the consecutive centralized market with the amount $\bar{m}\left(1-\bar{b}\right)$.

Using the envelope condition (5a), we can rewrite (11) to get

$$\Gamma^t(0) = \beta\,W^t_{+1}(0) + (1-\lambda)\,\gamma\,\beta\,\frac{\bar{m}\left(1-\bar{b}\right)}{p_{+1}} \tag{12}$$

An honest agent who meets a thief has the following payoff:

$$\Gamma^{h,t}(m,b) = \gamma\,\beta\,W^h\left[m\,b\,(1-\phi),0\right] + (1-\gamma)\,\beta\,W^h\left[m\,(1-b\,\phi),0\right] \tag{13}$$

With probability γ, the thief is successful and the honest agent looses his money except the deposits; with probability $(1-\gamma)$ he keeps his liquidity and goes to the next CM.

Let us finally turn to the decentralized market. The amount of dollars offered to a seller in exchange for the consumption good is denoted by d which cannot be greater than the money brought into the market, i.e. $d \leq m$. An honest buyer's payoff in the decentralized market then is

$$V^b(m,l) = u(q) + \beta\,W^h(m-d,l) \tag{14}$$

It consists of the utility from consuming the specialized good and the discounted continuation value for the consecutive CM.

An honest seller's payoff in the decentralized market is

$$V^s(m,\cdot) = -c(\bar{q}) + \beta\,W^h\left(m+\bar{d},\cdot\right) \tag{15}$$

An honest agent that is matched up with an agent of the same kind or with nobody gets the following payoff

$$V^n(m,\cdot) = \beta\, W^h(m,\cdot) \tag{16}$$

The marginal utility of having a bank loan for a buyer from (14) is

$$\frac{\partial V^b(m,l)}{\partial l} = \beta\,\frac{\partial W^h(\hat{m},l)}{\partial l} = -\beta\,\left(1+r^l\right)\frac{1}{p_{+1}} \tag{17}$$

where we used (5b) to get the last equation.

The marginal utility of money for a buyer in the DM from (14) is

$$\frac{\partial V^b(m,l)}{\partial m} = u'(q)\,\frac{\partial q}{\partial m} + \beta\,\frac{\partial W^h(\hat{m},l)}{\partial \hat{m}}\,\frac{\partial \hat{m}}{\partial m} \tag{18}$$

Together with the envelope conditions (5a) and (5b), (7a) can then be rewritten as

$$\frac{\partial \Gamma^b(m,b)}{\partial l} = u'(q)\,\frac{\partial q}{\partial m} + \frac{\beta}{p_{+1}}\,1(m>d) - \frac{\beta}{p_{+1}}\,\left(1+r^l\right) - \mu^b \tag{19}$$

Note that the middle term only applies if the agent is not cash constrained; otherwise it is equal to zero.

The marginal utility of money for a seller or a non-matched agent in the DM is

$$\frac{\partial V^j(m,\cdot)}{\partial m} = \beta\,\frac{\partial W^j(\hat{m},\cdot)}{\partial \hat{m}}\,\frac{\partial \hat{m}}{\partial m} = \frac{\beta}{p_{+1}} \tag{20}$$

what allows us to simplify (10) to get

$$\frac{\partial \Gamma^j(m,b)}{\partial b} = -m\,\phi\,\frac{\beta}{p_{+1}} + \mu^{j1} - \mu^{j2} \leq 0. \tag{21}$$

for all $j \in \{s,n\}$.

The terms of trade in the DM are determined as in Lagos and Wright (2005) using the generalized Nash bargaining solution, with $\theta > 0$ denoted as the bargaining power of a buyer. An honest buyer with m liquidity holdings and an honest seller with $m_s \in \{\bar{m},0\}$ settle on a pair (q_t,d_t) for all periods t that maximizes

$$\Big\{u(q) + \beta\,W^h[m-d,l] - \beta\,W^h[m,l]\Big\}^{\theta} \Big\{-c(q) + \beta\,W^h[m^s+d,\cdot] - \beta\,W^h[m^s,\cdot]\Big\}^{1-\theta}$$

subject to the buyers budget constraint $d \leq m$.

Using the envelope condition (5a), we can reduce the above problem to

$$\left[u\left(q\right)-\beta\,d\,\frac{1}{p_{+1}}\right]^{\theta}\,\left[-c\left(q\right)+\beta\,d\,\frac{1}{p_{+1}}\right]^{1-\theta} \tag{22}$$

Banks accept nominal deposits in the CM and the credit market at a fee ϕ for regular whole-period customers and at the operating cost for early withdrawers. In the credit market, banks offer loans to buyers to be redeemed in the consecutive CM at the nominal interest rate r^l. The operating costs for each bank are a per dollar on deposit and there are "reserve requirements" of measure ρ on the current deposits. The banking sector is perfectly competitive with free entry so banks make zero profit and take all interest rates (ϕ and r^l) as given. There is no strategic interaction among banks or between banks and agents. In particular, there is no bargaining over terms of the loan contract. A bank's profit consists of the interest rate on the outstanding loan, the fee for depositing money minus the cost of storing deposits. Additional to the profit on loans and deposits from non-buyers, banks get a fee payed on the deposits of buyers. But because we assume that buyers have to pay the operating cost a instead of ϕ when withdrawing cash, banks make no profits on this part. Hence, each bank solves the following problem per borrower:

$$\begin{aligned} &\max_{l}\ \left(\beta\,r^l+\phi-a\right)\,l \\ &\qquad \text{s.t}\quad l\le\bar{l}^b \\ \text{s.t}\quad &u\left(q\right)-\frac{\beta\,\left(1+r^l\right)\,l}{p_{+1}}\ge\Lambda \end{aligned} \tag{23}$$

where Λ is the reservation value of the borrower. Note that we have to discount the interest rate payment as it is due in the consecutive CM unlike the fee and the operating cost. This is the borrower's surplus from receiving a loan at another bank.

The first-order condition is

$$\beta\,r^l+\phi-a-\mu_B+\mu_\Lambda\,\left[u'\left(q\right)\,\frac{\partial\,q}{\partial\,m}\,\frac{\partial\,m}{\partial\,l}-\frac{\beta\,\left(1+r^l\right)}{p_{+1}}\right]=0 \tag{24}$$

where $\mu_B\ge 0$ and $\mu_\Lambda\ge 0$ are the Lagrange multipliers on the lending and the participation constraint of the borrower, respectively.

4 Equilibrium

Given the price levels of the CM we can write down the standard results as in Lagos and Wright (2005):

Lemma 1 *The solution to the bargaining problem* (22) *of the decentralized market is*

$$q = \begin{cases} g^{-1}\left(\frac{\beta}{p_{+1}} m\right) & \textit{if } m < m^* \\ q^* & \textit{if } m \geq m^* \end{cases}$$

$$d = \begin{cases} m & \textit{if } m < m^* \\ m^* & \textit{if } m \geq m^* \end{cases}$$

where $g(\cdot)$ *is given by*

$$g(q) \equiv \frac{\theta\, c(q)\ u'(q) + (1-\theta)\ u(q)\ c'(q)}{\theta\, u'(q) + (1-\theta)\ c'(q)}$$

The efficient quantity q^* *solves* $u'(q^*) = c'(q^*)$. *For the efficient money holdings we get*

$$m^* = g(q^*)\ \frac{p_{+1}}{\beta} \tag{25}$$

Proof Maximizing the bargaining problem (22) over q and d and subject to the buyer's budget constraints proves the Lemma. ■

Using the first-order conditions, we get the results for the centralized markets:

Lemma 2 *In equilibrium, each agent consumes the following quantities of the general good in the centralized markets:*

$$x = U'^{-1}(1)$$

Proof Follows directly from (4a). Same for thieves. ■

Lemma 3 *Honest agents that are not buyers will deposit all their cash on a bank account if the fee* ϕ *is negative.* □

Proof From (21) it follows that unless ϕ is negative, the marginal value of b (putting more money on the bank account) is negative. Hence, agents have maximum utility by not putting more money on the bank in this case. If ϕ is negative, the upper bound 1 will be reached, i.e. agents will deposit the remaining cash on the bank account to profit from an interest rate on deposits. ■

In case a buyer is not cash constrained, he will consume the efficient quantities. Otherwise, the marginal value of a loan (19) and the results from Lemma 1 imply

$$\frac{u'(q)}{g'(q)} = \left(1 + r^l\right) + \mu_b \frac{p_{+1}}{\beta} \tag{26}$$

where $\mu_b \geq 0$ is the Lagrange multiplier on the borrowing constraint. In case there is no borrowing constraint, e.g. with full enforcement, we get

$$\frac{u'(q)}{g'(q)} = \left(1 + r^l\right) \tag{27}$$

Hence, the buyer borrows up to the point where the marginal benefit of borrowing equals the marginal cost. An agent then spends all his money. Note that for $\left(r^l > 0\right)$ trades are inefficient, so a positive nominal interest rate acts as a tax on consumption. Finally, if an agent is constrained by his borrowing limit, we have

$$\frac{u'(q)}{g'(q)} > \left(1 + r^l\right) \tag{28}$$

The marginal value of an extra unit of loan then exceeds the marginal cost. Thus, a borrower would be willing to pay more than the prevailing loan rate. In case banks are worried about default, the interest rate may not rise to clear the market and credit rationing occurs. Consequently, all buyers borrow $\bar{l}^b$ and spend all their money to consume q.

The bank's first-order condition (24) together with the results from the bargaining become

$$\left(\beta\, r^l + \phi - a\right) - \mu_B + \mu_\Lambda \left[\frac{u'(q)}{g'(q)}\frac{\beta}{p_{+1}} - \frac{\beta\left(1 + r^l\right)}{p_{+1}}\right] = 0 \tag{29}$$

For $\left(\beta\, r^l + \phi - a\right) > 0$, the bank would like to make the largest loan possible to the borrower. Hence, she will always choose a loan size such that agents are indifferent between participating or not, i.e. $\mu_\Lambda > 0$. However, due to free entry, banks make zero profits such that $\beta\, r^l = (a - \phi)$. If there is no borrowing constraint, i.e. $\mu_B = 0$, the loan offered by the bank implies (27) which means that repayment is not an issue. If $\mu_B > 0$, the constraint on the loan size is binding and thus implies (28). In a symmetric equilibrium, all buyers borrow the same amount, l. The maximum possible amount that can be lend to agents, consists of the expected unused deposits. Thus, loan market clearing requires that

$$\overbrace{(1-\lambda)\,\sigma}^{\text{measure of buyers}}\, l = (1-\rho)\left[\underbrace{\lambda}_{\text{h meeting t}} + \underbrace{(1-\lambda)\,(1-\sigma)}_{\text{honest not being buyers}}\right]\underbrace{(1-\phi)\,m\,b}_{\text{deposits - fees}}$$
$$+ (1-\rho)\,(1-\lambda)\,(1-\sigma)\,(1-\phi)\,m\,(1-b)\,1\,(\phi < 0)$$

where the last term represents the remaining cash that is deposited on the bank account by sellers and non-matching agents in case they get a negative fee ϕ (see Lemma 3).

This reduces to

$$(1-\lambda)\,\sigma\, l = (1-\rho)\,[1 - \sigma\,(1-\lambda)]\,(1-\phi)\,m\,b \quad (\text{if } \phi \geq 0) \tag{30a}$$

$$(1-\lambda)\,\sigma\, l = (1-\rho)\,[\lambda\, b + (1-\lambda)\,(1-\sigma)]\,(1-\phi)\,m \quad (\text{if } \phi < 0) \tag{30b}$$

These results let us state the following

Lemma 4 *In equilibrium, banks make zero profit and the loan market clears. Hence,*

$$r^l = (a - \phi)\, \frac{1}{\beta}$$
$$l = \frac{(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)}{(1-\lambda)\,\sigma}\, m\, b \quad \textit{if}\, \phi \geq 0$$
$$l = \frac{(1-\rho)\,[\lambda\, b + (1-\lambda)\,(1-\sigma)]\,(1-\phi)}{(1-\lambda)\,\sigma}\, m \quad \textit{if}\, \phi < 0$$

Proof From the above discussion. ∎

One important implication from Lemma 4 is the fact that private bank loans can only exist, when private banks exist and money is taken out of the CM.

As described in Section 2, we assume for simplicity that the government wants to keep its budget equal to zero over the time. This means that the government sterilizes the deflationary effect of the interest payment. Hence,

Lemma 5 *The equilibrium lump-sum transfers by definition are*

$$T = \pi_{+1}\, M$$

Differentiating (6), (8), (9), and (13) with respect to m, we get the following marginal values of money in the credit market:

Lemma 6 *Let $L(q) \equiv u'(q)/g'(q) - 1$. Then*

$$m \geq m^* \Rightarrow \frac{\partial \Gamma^b(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - b\,a) - \frac{1}{p_{+1}}(a - \phi)\frac{(1-\rho)[1-\sigma(1-\lambda)](1-\phi)}{(1-\lambda)\sigma}b \quad \forall \phi \geq 0$$

$$m \geq m^* \Rightarrow \frac{\partial \Gamma^b(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - b\,a) - \frac{1}{p_{+1}}(a - \phi)\frac{(1-\rho)[\lambda b + (1-\lambda)(1-\sigma)](1-\phi)}{(1-\lambda)\sigma} \quad \forall \phi < 0$$

$$m < m^* \Rightarrow \frac{\partial \Gamma^b(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - b\,a) - \frac{1}{p_{+1}}(a - \phi)\frac{(1-\rho)[1-\sigma(1-\lambda)](1-\phi)}{(1-\lambda)\sigma}b + \frac{\beta}{p_{+1}}\frac{(1-\lambda)\sigma(1-b\,a) + (1-\rho)[1-\sigma(1-\lambda)](1-\phi)\,b}{(1-\lambda)\sigma}L(q) \quad \forall \phi \geq 0$$

$$m < m^* \Rightarrow \frac{\partial \Gamma^b(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - b\,a) - \frac{1}{p_{+1}}(a - \phi)\frac{(1-\rho)[\lambda b + (1-\lambda)(1-\sigma)](1-\phi)}{(1-\lambda)\sigma} + \frac{\beta}{p_{+1}}\frac{(1-\lambda)\sigma(1-b\,a) + (1-\rho)[\lambda b + (1-\lambda)(1-\sigma)](1-\phi)}{(1-\lambda)\sigma}L(q) \quad \forall \phi < 0$$

$$\frac{\partial \Gamma^s(m,b)}{\partial m} = \frac{\partial \Gamma^n(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - b\,\phi) \quad \forall \phi \geq 0$$

$$\frac{\partial \Gamma^s(m,b)}{\partial m} = \frac{\partial \Gamma^n(m,b)}{\partial m} = \frac{\beta}{p_{+1}}(1 - \phi) \quad \forall \phi < 0$$

$$\frac{\partial \Gamma^{h,t}(m,b)}{\partial m} = \frac{\beta}{p_{+1}}[\gamma\, b\,(1-\phi) + (1-\gamma)(1 - b\,\phi)]$$

Proof Take the derivatives of the value functions from the credit market with respect to m and use the implicit function theorem to get $\partial q/\partial m$ and the results from Lemma 4.■

Let us now, as in He, Huang, and Wright (2008), put our results in context to monetary policy and particularly to the nominal interest rate. We define i to be the nominal interest rate between two successive openings of the CM. From (1), it follows that i satisfies the following condition

$$1+i=\frac{p_{+1}}{p}\frac{1}{\beta} \tag{31}$$

Note that in standard models, a monetary equilibrium only exists if $i \geq 0$. This however does not necessarily have to be the case here, as shown in the following

Lemma 7 *A monetary equilibrium exists iff $i \geq i^*$, where*

$$\begin{aligned} i^*_{\phi\geq 0} \equiv \Big\{ & 1-\lambda\gamma\ (1-b)-\phi\, b-(1-\lambda)\ \sigma\, b\ (a-\phi) \\ & -\frac{1}{\beta}\ (1-\rho)\ (a-\phi)\ (1-\phi)\ [1-(1-\lambda)\ \sigma]\ b\Big\}-1 \end{aligned} \tag{32a}$$

$$\begin{aligned} i^*_{\phi<0} \equiv \Big\{ & 1-\phi-(1-\lambda)\ \sigma\, b\, a-(1-b)\ \gamma\,\lambda+\phi\ [\sigma+\lambda\ (1-b-\sigma)] \\ & -\frac{1}{\beta}\ (1-\rho)\ (a-\phi)\ (1-\phi)\ [\lambda\, b+(1-\lambda)\ (1-\sigma)]\Big\}-1 \end{aligned} \tag{32b}$$

Proof Use the first-order conditions (4b), the results from Lemma 6 updated by one period, the marginal change of the loan with respect to m from Lemma 4, and the Fisher equation (31). ■

Given $i > i^*$, the first-order conditions for $\hat{m}$ imply:

$$\begin{aligned} L\,(q) = & \frac{i+\lambda\gamma\ (1-b)+\phi\, b+(1-\lambda)\ \sigma\, b\ (a-\phi)}{(1-\lambda)\ \sigma\ (1-b\, a)+(1-\rho)\ [1-\sigma\ (1-\lambda)]\ (1-\phi)\ b} \\ & + \frac{\frac{1}{\beta}\ (1-\rho)\ (a-\phi)\ (1-\phi)\ [1-(1-\lambda)\ \sigma]\ b}{(1-\lambda)\ \sigma\ (1-b\, a)+(1-\rho)\ [1-\sigma\ (1-\lambda)]\ (1-\phi)\ b} \quad \forall\, \phi \geq 0 \end{aligned} \tag{33a}$$

$$\begin{aligned} L\,(q) = & \frac{i+\phi+(1-\lambda)\ \sigma\, b\, a+(1-b)\ \gamma\,\lambda-\phi\ [\sigma+\lambda\ (1-b-\sigma)]}{(1-\lambda)\ \sigma\ (1-b\, a)+(1-\rho)\ [\lambda\, b+(1-\lambda)\ (1-\sigma)]\ (1-\phi)} \\ & + \frac{\frac{1}{\beta}\ (1-\rho)\ (a-\phi)\ (1-\phi)\ [\lambda\, b+(1-\lambda)\ (1-\sigma)]}{(1-\lambda)\ \sigma\ (1-b\, a)+(1-\rho)\ [\lambda\, b+(1-\lambda)\ (1-\sigma)]\ (1-\phi)} \quad \forall\, \phi < 0 \end{aligned} \tag{33b}$$

In order to derive the equilibrium measure of b, let us first state the marginal values of bank deposits in the credit market:

Lemma 8

$$m \geq m^* \Rightarrow \frac{\partial\,\Gamma^b\,(m,b)}{\partial\,b} = -\frac{\beta}{p_{+1}}\,m\,a - \frac{1}{p_{+1}}\,(a-\phi)\,\frac{(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)}{(1-\lambda)\,\sigma}\,m \quad \forall\,\phi \geq 0$$

$$m \geq m^* \Rightarrow \frac{\partial\,\Gamma^b\,(m,b)}{\partial\,b} = -\frac{\beta}{p_{+1}}\,m\,a - \frac{1}{p_{+1}}\,(a-\phi)\,\frac{(1-\rho)\,\lambda\,(1-\phi)}{(1-\lambda)\,\sigma}\,m \quad \forall\,\phi < 0$$

$$m < m^* \Rightarrow \frac{\partial\,\Gamma^b\,(m,b)}{\partial\,b} = -\frac{\beta}{p_{+1}}\,m\,a - \frac{1}{p_{+1}}\,(a-\phi)\,\frac{(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\,m}{(1-\lambda)\,\sigma} + \frac{\beta}{p_{+1}}\frac{-(1-\lambda)\,\sigma\,m\,a+(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\,m}{(1-\lambda)\,\sigma}\,L\,(q) \quad \forall\,\phi \geq 0$$

$$m < m^* \Rightarrow \frac{\partial\,\Gamma^b\,(m,b)}{\partial\,b} = -\frac{\beta}{p_{+1}}\,m\,a - \frac{1}{p_{+1}}\,(a-\phi)\,\frac{(1-\rho)\,\lambda\,(1-\phi)\,m}{(1-\lambda)\,\sigma} + \frac{\beta}{p_{+1}}\frac{-(1-\lambda)\,\sigma\,m\,a+(1-\rho)\,\lambda\,(1-\phi)\,m}{(1-\lambda)\,\sigma}\,L\,(q) \quad \forall\,\phi < 0$$

$$\frac{\partial\,\Gamma^s\,(m,b)}{\partial\,b} = \frac{\partial\,\Gamma^n\,(m,b)}{\partial\,b} = -\frac{\beta}{p_{+1}}\,m\,\phi \quad \forall\,\phi \geq 0$$

$$\frac{\partial\,\Gamma^s\,(m,b)}{\partial\,b} = \frac{\partial\,\Gamma^n\,(m,b)}{\partial\,b} = 0 \quad \forall\,\phi < 0$$

$$\frac{\partial\,\Gamma^{h,t}\,(m,b)}{\partial\,b} = \frac{\beta}{p_{+1}}\,m\,(\gamma-\phi)$$

Proof Take the derivatives of the credit market value functions with respect to b and use the implicit function theorem to get $\partial\,q/\partial\,m$, and the results from Lemma 4. ■

Using the first-order conditions for the choice of bank deposits in the CM, (4c), we get the following equation

$$0 = \frac{\beta}{p_{+1}}\Big(\lambda\,\gamma-\phi-\sigma\,(1-\lambda)\,(a-\phi)-\frac{1}{\beta}\,(a-\phi)\,(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi) + \{-a\,(1-\lambda)\,\sigma+(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\}\,L\,(q)\Big)\,m \quad (34)$$

for $\phi \geq 0$ and

$$0 = \frac{\beta}{p_{+1}} \Big(\lambda\,\gamma - \lambda\,\phi - \sigma\,(1-\lambda)\,a - \frac{1}{\beta}\,(a-\phi)\,(1-\rho)\,\lambda\,(1-\phi) \\ + \{-a\,(1-\lambda)\,\sigma + (1-\rho)\,\lambda\,(1-\phi)\}\,L(q) \Big)\,m \quad (35)$$

for $\phi < 0$.

Thus, the first-order condition from the CM implies for $m < m^*$ and $\phi \geq 0$

$$b \begin{cases} = 1 \\ \in (0,1) \\ = 0 \end{cases} \quad \text{if} \quad \lambda \gtreqless \\ \frac{\beta\,\phi + \beta\,\sigma\,(a-\phi) + (a-\phi)\,(1-\phi)\,(1-\rho)\,(1-\sigma)}{\beta\,\gamma + \beta\,\sigma\,(a-\phi) - (a-\phi)\,(1-\phi)\,(1-\rho)\,\sigma + \beta\,L(q)\,[a\,\sigma + (1-\rho)\,\sigma\,(1-\phi)]} \\ - \frac{\beta\,L(q)\,[-a\,\sigma + (1-\rho)\,(1-\sigma)\,(1-\phi)]}{\beta\,\gamma + \beta\,\sigma\,(a-\phi) - (a-\phi)\,(1-\phi)\,(1-\rho)\,\sigma + \beta\,L(q)\,[a\,\sigma + (1-\rho)\,\sigma\,(1-\phi)]} \quad (36)$$

and for $m \geq m^*$ and $\phi \geq 0$

$$b \begin{cases} = 1 \\ \in (0,1) \\ = 0 \end{cases} \quad \text{if} \quad \lambda \gtreqless \frac{\beta\,\phi + \beta\,\sigma\,(a-\phi) + (a-\phi)\,(1-\phi)\,(1-\rho)\,(1-\sigma)}{\beta\,\gamma + \beta\,\sigma\,(a-\phi) - (a-\phi)\,(1-\phi)\,(1-\rho)\,\sigma} \quad (37)$$

In the case where the banking fee is negative, $\phi < 0$, we similarly get for $m < m^*$

$$b \begin{cases} = 1 \\ \in (0,1) \\ = 0 \end{cases} \quad \text{if} \quad \lambda \gtreqless \frac{a\,\sigma + a\,\sigma\,L(q)}{\gamma - \phi + \sigma\,a - \frac{1}{\beta}\,(a-\phi)\,(1-\phi)\,(1-\rho) + L(q)\,[a\,\sigma + (1-\rho)\,(1-\phi)]} \quad (38)$$

and for $m \geq m^*$ and $\phi < 0$

$$b \begin{cases} = 1 \\ \in (0,1) \\ = 0 \end{cases} \quad \text{if} \quad \lambda \gtreqless \frac{a\,\sigma}{\gamma - \phi + a\,\sigma - \frac{1}{\beta}\,(a-\phi)\,(1-\phi)\,(1-\rho)} \quad (39)$$

Our next step is to compare the values of becoming an honest agent or a thief. Using the value functions in the credit market and the decentralized market together with the value function of the centralized market, (3), and the envelope conditions, (5), yields for an honest agent in steady state the value of a period with zero initial money holdings and zero outstanding loans.

$$(1-\beta)\ W^h(0,0) = U(x) - x + \frac{T-\hat{m}}{p} + (1-\lambda)\ \sigma\ [u(q) - c(q)]$$
$$+\frac{\beta}{p_{+1}}\ [1-\lambda\gamma\ (1-b) - \phi b - (1-\lambda)\ \sigma b\ (a-\phi)]\ \hat{m} - \frac{\beta}{p_{+1}}\ (1-\lambda)\ \sigma\, l\, r^l \quad \forall \phi \geq 0 \tag{40}$$

and

$$(1-\beta)\ W^h(0,0) = U(x) - x + \frac{T-\hat{m}}{p} + (1-\lambda)\ \sigma\ [u(q) - c(q)]$$
$$+\frac{\beta}{p_{+1}}\ \{1-\phi-(1-\lambda)\ \sigma b a - (1-b)\ \gamma\lambda + \phi\ [\sigma + \lambda\ (1-b-\sigma)]\}\ \hat{m}$$
$$-\frac{\beta}{p_{+1}}\ (1-\lambda)\ \sigma\, l\, r^l \quad \forall \phi < 0 \tag{41}$$

Note that we explicitly want to compare an honest agent and a thief who start without any outstanding loans, what guarantees full enforcement of repayment. (Including repayment of an outstanding loan in the honest agents problem but not in the thief's problem could otherwise been interpreted as becoming a thief means escaping the loan repayment.)

Similarly, from (2) and (11) we get

$$(1-\beta)\ W^t(0) = U(x) - x + \frac{T}{p} + \beta\gamma\ (1-\lambda)\ \frac{\hat{\hat{m}}\ \left(1-\bar{b}\right)}{p_{+1}} \tag{42}$$

As there cannot be any monetary equilibrium with $\lambda = 1$, we will focus on equilibria where $\lambda \in [0,1)$. Let us define the "honesty premium"

$$\Delta(m) = W^h(m) - W^t(m) \tag{43}$$

Note that because of the linearity in m of W^t and W^h, Δ does not depend on m.

An equilibrium requires that

- $\lambda = 0$ if $\Delta > 0$
- $\Delta = 0$ if $\lambda \in (0,1)$

In other words, this condition tells us that it will not be optimal for an agent to become a thief, if it is not equally valuable as being an honest person.

The difference between (40) and (42) then yields

$$(1-\beta)\ \Delta_{\phi\geq 0} = -\frac{\hat{m}}{p} + (1-\lambda)\ \sigma\ [u(q) - c(q)]$$
$$+ [1 - \gamma\ (1-b) - \phi\, b - (1-\lambda)\ \sigma\, b\ (a-\phi)]\ \hat{m}\,\frac{\beta}{p_{+1}} - \frac{\beta}{p_{+1}}\ (1-\lambda)\ \sigma\, l\, r^l$$

$$(1-\beta)\ \Delta_{\phi<0} = -\frac{\hat{m}}{p} + (1-\lambda)\ \sigma\ [u(q) - c(q)]$$
$$+\{1 - \phi - (1-\lambda)\ \sigma\, b\, a - (1-b)\ \gamma + \phi\ [\sigma + \lambda\ (1-b-\sigma)]\}\ \hat{m}\,\frac{\beta}{p_{+1}} - \frac{\beta}{p_{+1}}\ (1-\lambda)\ \sigma\, l\, r^l$$

Rearranging using the bargaining solution from Lemma 1, Lemma 4, the Fisher equation, and the budget constraints from the credit market allows us to get the "honesty premium" depending entirely on the quantities traded in the DM:

$$\begin{aligned}(1-\beta)\ \Delta_{\phi\geq 0} &= (1-\lambda)\,\sigma\ [u(q) - c(q)] \\ &- \frac{[i + \gamma\ (1-b) + \phi\, b + (1-\lambda)\ \sigma\, b\ (a-\phi)]\ (1-\lambda)\ \sigma}{(1-a\,b)\ (1-\lambda)\ \sigma + (1-\rho)\ [1-\sigma\ (1-\lambda)]\ (1-\phi)\ b}\, g(q) \\ &- \frac{\frac{1}{\beta}\ (1-\lambda)\ \sigma\ (a-\phi)\ (1-\rho)\,[1-\sigma\ (1-\lambda)]\ (1-\phi)\ b}{(1-a\,b)\ (1-\lambda)\ \sigma + (1-\rho)\ [1-\sigma\ (1-\lambda)]\ (1-\phi)\ b}\, g(q)\end{aligned} \quad (44)$$

$$\begin{aligned}(1-\beta)\ \Delta_{\phi<0} &= (1-\lambda)\,\sigma\ [u(q) - c(q)] \\ &- \frac{\{i + \phi + (1-\lambda)\ \sigma\, b\, a + (1-b)\ \gamma - \phi\ [\sigma + \lambda\ (1-b-\sigma)]\}\ (1-\lambda)\ \sigma}{(1-a\,b)\ (1-\lambda)\ \sigma + (1-\rho)\ [\lambda\, b + (1-\lambda)\ (1-\sigma)]\ (1-\phi)}\, g(q) \\ &- \frac{\frac{1}{\beta}\ (1-\lambda)\ \sigma\ (a-\phi)\ (1-\rho)\ [\lambda\, b + (1-\lambda)\ (1-\sigma)]\ (1-\phi)}{(1-a\,b)\ (1-\lambda)\ \sigma + (1-\rho)\ [\lambda\, b + (1-\lambda)\ (1-\sigma)]\ (1-\phi)}\, g(q)\end{aligned} \quad (45)$$

4.1 Banking without Loans

Let us now first analyze an equilibrium with banks that are not allowed to issue loans, as in He, Huang, and Wright (2008). Therefore, we assume in this section that $\rho = 1$. Furthermore, as we have perfect competition between banks it follows that $a = \phi$.

Because $\lambda \in [0, 1]$, a necessary condition for an equilibrium is the assumption of

$$\gamma \geq a > 0$$

which follows directly from (36).

From the above condition and (36) it follows that

$$\lambda \to 0 \quad \text{as} \quad a \to 0$$

This means that a reduction in the deposit cost a will reduce the amount of cash agents take into the credit market. However, our condition shows that the probability of a successful theft tends towards zero as well. As such a case would on the opposite drive out bank deposits of the market, we expect that it is not possible to have a cashless economy with λ endogenous.

There are three possible candidates for an equilibrium, i.e. $b = 1$, $b = 0$, and $b \in (0, 1)$. As suggested above, we cannot have $b = 1$ as long λ is endogenous. Let us therefore analyze equilibria where $b \in [0, 1)$.

If and only if $\lambda\,\gamma/\left[1 + \sigma\ (1 - \lambda)\ L\,(q)\right] \leq a$ from (34), $b = 0$ is a best response. As this means that we do not have any banks nor bank loans but endogenous theft, q and λ are determined exactly as in He, Huang, and Wright (2006, Section 4). Let us repeat the relevant parts of their Proposition 2: $\lambda \in (0, 1)$ implies that $q = \tilde{q}$ is independent from i where $\tilde{q}$ is the quantity consumed when there are no banks but endogenous thieves, i.e.

$$L\,(\tilde{q}) = \frac{\sigma\,[u\,(\tilde{q}) - c\,(\tilde{q})] - \gamma\,g\,(\tilde{q})}{\sigma\,g\,(\tilde{q})} \tag{46}$$

from (33a) and (44), as long as $i > -\gamma = i^*$ (otherwise we would divide by zero to get this result). Further, $\lambda \in (0, 1)$ iff $i < i^0$ where

$$i^0 = \frac{\sigma\ [u\,(\tilde{q}) - c\,(\tilde{q})]}{g\,(\tilde{q})} - \gamma \tag{47}$$

from (44). If $i^0 < 0$ then $i^* = 0$ and $\lambda = 0$; and if $i^0 > 0$ then $i^* = -\gamma$, in which case $\lambda > 0$ for $i \in [i^*, i^0)$ and $\lambda = 0$ for $i \geq i^0$. Given this, we can check the best response condition. If $\lambda = 0$, then $\lambda\,\gamma \leq a\ [1 + \sigma\ (1 - \lambda)\ L\,(q)]$ and $b = 0$ is obviously the best response. The most interesting case is $i^0 > 0$ and $i \in [i^*, i^0)$, where $\lambda \in (0, 1)$ is equal to

$$\tilde{\lambda} = 1 - \frac{(i + \gamma)\ g\,(\tilde{q})}{\sigma\ [u\,(\tilde{q}) - c\,(\tilde{q})]} \tag{48}$$

from (44). In this case, as q is independent of b, $\lambda\,\gamma \leq a$ iff $i \geq i^1$, where

$$i^1 \equiv \frac{\gamma\ (1 - a)\ \sigma\ [u\,(\tilde{q}) - c\,(\tilde{q})]}{g\,(\tilde{q})\ \gamma\ (1 - a) + a\,\sigma\ [u\,(\tilde{q}) - c\,(\tilde{q})]} - \gamma \tag{49}$$

from (34), (44), and (46). Note that this interest rate is slightly different from the solution in He, Huang, and Wright (2006).

We conclude that when $i^0 > 0$ and $i \in [i^*, i^0)$, $b = 0$ is an equilibrium iff $i \geq i^1$.

As i decreases further than i^1, we get $\lambda\gamma > a\,[1 + \sigma\,(1-\lambda)\,L(q)]$ and the best response to that becomes $b = 1$. But $b = 1$ cannot be an equilibrium, hence the only sensible thing is to stay at $\lambda\gamma = a\,[1 + \sigma\,(1-\lambda)\,L(q)]$ when i falls below i^1, so that $b \in (0,1)$ is a best response; and b adjusts so that $\lambda\gamma \leq a\,[1 + \sigma\,(1-\lambda)\,L(q)]$ is a best response. Therefore, a monetary equilibrium for $i \in [i^*, i^1)$ is determined by the following three conditions:

(i) $b \in (0,1)$ is a best response, which means from (36)

$$\lambda = \frac{a\,[1 + \sigma\,L(q)]}{\gamma + a\,\sigma\,L(q)}$$

(ii) $\lambda \in (0,1)$ is a best response, so $\Delta = 0$, or from (44)

$$\lambda = 1 - \frac{[i + \gamma\,(1-b) + b\,a]\;g(q)}{(1 - b\,a)\;\sigma\;[u(q) - c(q)]} \tag{50}$$

(iii) q satisfies condition (33a).

In order to maintain $0 < b < 1$, we need to have $\lambda = a\,[1 - \sigma\,L(q)]\,/\,[\gamma + a\,\sigma\,L(q)] < 1$, i.e. $a < \gamma$. In this case, as (32a) together with (33a) and (36)

$$i^* = -\frac{a\,\gamma + [b\,a + (1-b)\,\gamma]\;a\,\sigma\,L(q)}{\gamma + a\,\sigma\,L(q)} = -a \tag{51}$$

(49) implies that $i \in (i^*, i^0)$. Given $i \in [i^*, i^1)$, this equilibrium exists and, using (33a) and (36) we get

$$L(q) = \frac{\gamma\,(a+i)}{[\gamma\,(1-a) - a\,(1+i)]\;\sigma} \tag{52}$$

which can be solved for q with $\partial q/\partial i < 0$ under our assumption that $a < \gamma$.

Finally, using (50) we get an expression for b:

$$b = \frac{-\gamma\,(\gamma+i)\,(1-a)\;g(q) + [\gamma - a - (\gamma+i)\,a]\;\sigma\;[u(q) - c(q)]}{-\gamma\,(\gamma-a)\,(1-a)\;g(q) + a\,[\gamma - a - (\gamma+i)\,a]\;\sigma\;[u(q) - c(q)]} \tag{53}$$

Comparing these results to the original results from He, Huang, and Wright (2006) we see that there are significant differences. In contrast to them, we take the bank fees in the cash constraint of the buyers into account rather than solving for the unconstrained model. Hence, λ is not constant in the bank case.

Figure 2 depicts the equilibria. Note that this figure looks almost exactly as the figure in He, Huang, and Wright (2006) with the major difference being λ not constant in the bank case.

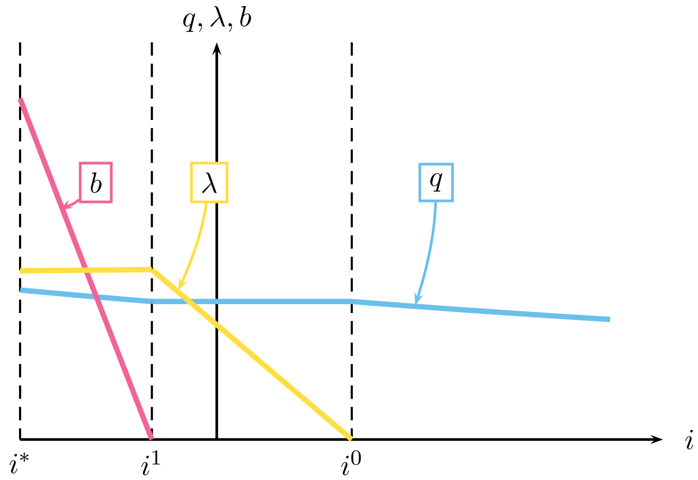

Figure 2: Banking without loans

4.2 Banking with Loans

In this section, we now allow loans from private banks. Let us therefore assume $\rho = 0$. We then solve for the equilibria as in the previous section. Further, note that we only present the equilibrium solution when non-buyers do not readjust their bank deposits in the credit market. The solution with readjustment is analogous using the respective equilibrium conditions from this Section.

For $\lambda \in [0,1]$ to hold, a necessary condition for an equilibrium is the assumption of $[a - \beta L(q) - \phi]\,(1-\phi) + \beta\phi \le \beta\gamma$ which follows directly from (36). Second, it follows from the same equation that $\lambda \to 0$ and $\phi \to 0$ as long as $a \ge \beta L(q)$ in equilibrium. (In an equilibrium with banks but no thieves this condition inverted.)

This condition guarantees that a reduction in the deposit cost ϕ will reduce the amount of cash agents take into the DM as well as the expected cost of holding money. As such a case would on the opposite drive out bank deposits from the market, we expect that it is not possible to have a cashless economy with λ endogenous. Although, there may exist an equilibrium where no agent wants to become a thief but where still bank deposits exist due to the fact that the bank may finance a positive interest rate on deposits through the loans that buyers acquire in the credit market.

Again, there are three possible candidates for an equilibrium, i.e. $b = 1$, $b = 0$, and $b \in (0,1)$. As suggested above, we cannot have $b = 1$ as long as λ is endogenous under the above conditions. Let us therefore analyze equilibria where $b \in [0,1)$.

The case $b = \lambda = 0$ is solved exactly the same way as in Section 4.1. To recapitulate, there exists an equilibrium threshold interest rate i^0 above which no banks and no thieves operate anymore. Whereas in the case without loans we have an equilibrium with coexistence of banks and thieves as long as the interest rate is low, i.e. between i^* and i^1. Second, between i^1 and i^0 there exists an equilibrium without banks but with thieves. The reason for that is the fact that the fees generated by the banks' business to offer deposit facilities are defined by the competitive market of banks and the exogenous operating cost a. Whereas agents already have to bear the higher inflation cost of $i \in [i^1, i^0]$ depositing money additionally poses some cost which, together with the decreasing measure of thieves, makes it unattractive to deposit money in a private bank. Now, with bank loans, things change completely as agents have a benefit now from the existence of banks. An increase in the interest rates will still make it unattractive for agents to become thieves as they do not have any possibility to escape the inflation tax. However, depositing money will be attractive up to $i = i^0$ even without thieves due to the interest rates ϕ payed on deposits.

We will first analyze an equilibrium with banks but without thieves and then conclude with the equilibrium with banks and thieves.

Note that we assume full enforcement of bank loan repayment, hence agents do not have a borrowing constraint given by the bank, i.e. $\mu_B = 0$ from (24). As there is perfect competition among banks, each bank makes zero profit. It follows thus from (24) that $\phi = a - \beta\, L(q)$.

From this equality and the fact that there are no thieves ($\lambda = 0$), we solve (33a) to get an expression for $b = \breve{b}$:

$$\breve{b}_{\phi \geq 0} = \frac{\sigma\, L(\breve{q}) - i}{[a - \beta\, L(\breve{q})] + (a + \beta)\ \sigma\, L(\breve{q})} \tag{54}$$

where we denote $q = \breve{q}$ as the quantity produced in the case with banks but without thieves.

Using this $\breve{b}$ from above together with the definition of ϕ and $\lambda = 0$, we can solve the "honesty premium" (44) for $\breve{q}$. Note that the honesty premium still has to be equal to zero in this case as we only have zero thieves in equilibrium (in fact however, agents still are indifferent between becoming a thief or not).

Solving the "honesty premium" (44) for $b = \breve{b}$ has the illustrative effect that it is immediately clear from inspection of

$$\breve{b} = \{(\gamma + i)\ g(\breve{q}) - \sigma\ [u(\breve{q}) - c(\breve{q})]\} / (\dots)$$

that $\breve{b}$ becomes zero at $i = i^0$ from (47).

We get the lower bound of the interest i that yields an equilibrium without thieves but with banks by solving (33a) for i together with the definition of ϕ and the fact that $\lambda = 0$:

$$i^2 = \frac{a\,\sigma}{\beta\ (1 - \sigma) - a\,\sigma} \tag{55}$$

Let us conclude the case of banks without thieves by summarizing our results in the following

Lemma 9 *With endogenous λ and banks, there exists an equilibrium with $\lambda = 0$, $b = \breve{b} \in (0, 1)$, and $q = \breve{q}$ for any $i \in (i^2, i^0]$.* □

Figure 3 shows a numerical example of this Lemma using the same numbers as in the case of He, Huang, and Wright (2008).

For i below i^2, $\lambda \in (0, 1)$ is a best response; and λ adjusts so that b is a best response. Therefore, a monetary equilibrium for $i \in [i^*, i^2]$ is determined by the following four conditions:

(i) $\phi = a - \beta\, r^l$ from the zero profit condition of the banks and the assumption of full enforcement that leads to (27).

(ii) $b \in (0, 1)$ is a best response, which means from (36)

$$\lambda = \frac{a\ [1 + \sigma\, L(q)] - \beta\, L(q)\ (1 - \sigma)}{\gamma + (a + \beta)\ \sigma\, L(q)}$$

(iii) $\lambda \in (0,1)$ is a best response, which means $\Delta = 0$, or from (44)

$$0 = (1-\lambda)\,\sigma\,[u(q) - c(q)] - \frac{[i + \gamma\,(1-b) + \phi\,b + (1-\lambda)\,\sigma\,b\,(a-\phi)]\,(1-\lambda)\,\sigma}{(1-a\,b)\,(1-\lambda)\,\sigma + (1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\,b}\,g(q) - \frac{\frac{1}{\beta}\,(1-\lambda)\,\sigma\,(a-\phi)\,(1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\,b}{(1-a\,b)\,(1-\lambda)\,\sigma + (1-\rho)\,[1-\sigma\,(1-\lambda)]\,(1-\phi)\,b}\,g(q) \quad (56)$$

(iv) q satisfies the usual condition (33a).

As there are four equations and four unknowns, namely ϕ, q, b, λ, we get an equilibrium. All other variables, like m etc., follow immediately from the other conditions in this Section.

Let us finally solve for the lower bound of the interest rate when $0 < b < 1$ and $\lambda \in [0,1)$. In case the interest rate i drops below that lower bound rate i^*, we do not have a monetary equilibrium anymore. So, using (32a) together with (33a) yields

$$i^* = -[1 - L(q)] - L(q)\,[1 - \beta + \beta\,L(q)] = -a$$

and $L(q) = (a+i)/\beta$. So, the lower bound of the monetary equilibrium with bank loans is exactly the same lower bound rate as in the case without private bank loans, cf. (51).

Let us state

Lemma 10 *With endogenous λ and banks, there exists an equilibrium with $\lambda \in (0,1)$, $b = \in (0,1)$, and q for any $i \in [i^*, i^2]$.* □

Figure 3 presents the equilibrium values of b, q, and λ in the case where banks may issue private loans. Numerically, we have used exactly the same parameters as in Figure 2 what allows us to compare the two equilibria nicely. (Note that these graphs show the version where non-buyers do not readjust their deposits when $\phi < 0$.) First of all, we recognize that the lower bound interest rates i^* are the same for both equilibria. Second, the upper bound interest rate below either banks or thieves exist, i^0, are the same as well. In between, banks will survive higher interest rates than in the case where they cannot issue private loans. They will even survive thieves, what means that banks exist in equilibrium even if it is in expectation completely costless for any agent to hold cash. One obvious reason for that is the fact that banks give agents a benefit in the form of being a lender of last resort for buyers in the decentralized market. Second, note that $\phi = 0$ at $i = 0$. Hence, there exists an equilibrium with and without thieves, where banks will pay an interest rate on deposits. As q further decreases with i, it follows from the definition of $L(q)$ that $\phi = a - \beta\,L(q)$ decreases and thus becomes negative. See Figure 4 for an illustration.

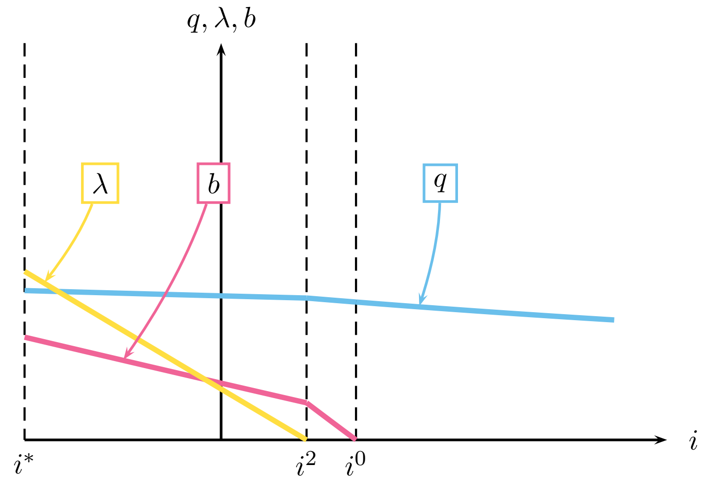

Figure 3: Banking with loans

5 Welfare and the Multiplier

After having characterized the equilibria, we now want to characterize welfare. A sensible welfare criterion is an honest agent's value at the beginning of the centralized market entering without any money and without any outstanding loans. Note that because we only analyze welfare in a monetary equilibrium, this welfare criterion is equivalent to the value of a thief because of the "honesty premium" being equal to zero. Hence, let us repeat (40) for better understanding:

$$\mathcal{W} = (1-\beta)\ W^h\,(0,0) = U\,(x) - x + \frac{T-\hat{m}}{p} + (1-\lambda)\ \sigma\ [u\,(q) - c\,(q)]$$
$$+\frac{\beta}{p_{+1}}\ [1 - \lambda\,\gamma\ (1-b) - \phi\,b - (1-\lambda)\ \sigma\,b\ (a-\phi)]\ \hat{m} - \frac{\beta}{p_{+1}}\ (1-\lambda)\ \sigma\,l\,r^l \quad \forall\,\phi \geq 0$$

Using the results from the previous Sections, we plot the welfare for the case with and without loans in Figure 5.

The money multiplier tells us how many dollars are generated by bringing m into the credit market. In other words, it gives us an indicator on the benefit of the banking system.

$$\mathcal{M} = \frac{m\ (1-a\,b) + l}{m} \tag{57}$$

As the deposits are endogenous, we get a multiplier that also depends on the interest rate. Figure 6 depicts the multiplier as a function of i.

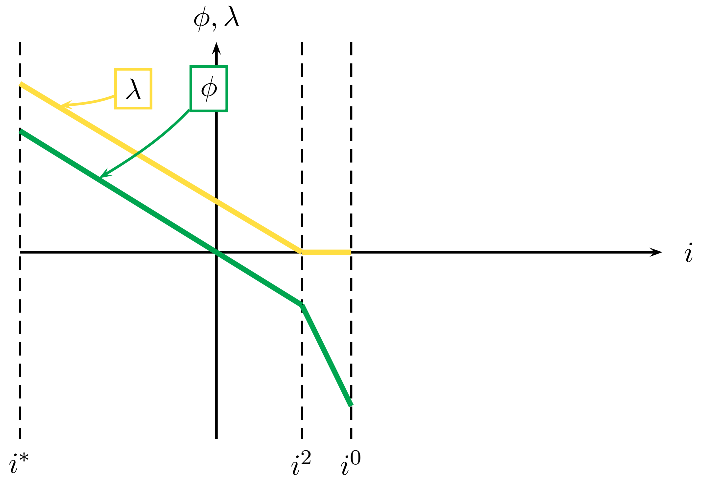

Figure 4: The deposit fee/interest rate ϕ in the model with bank loans.

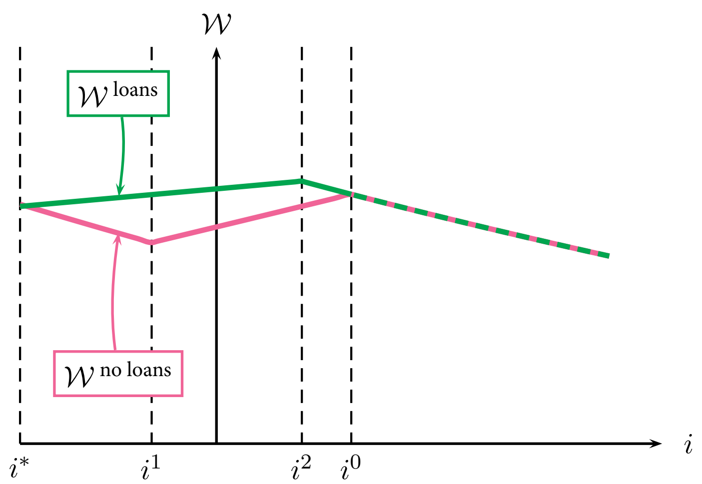

Figure 5: Welfare (with and without loans)

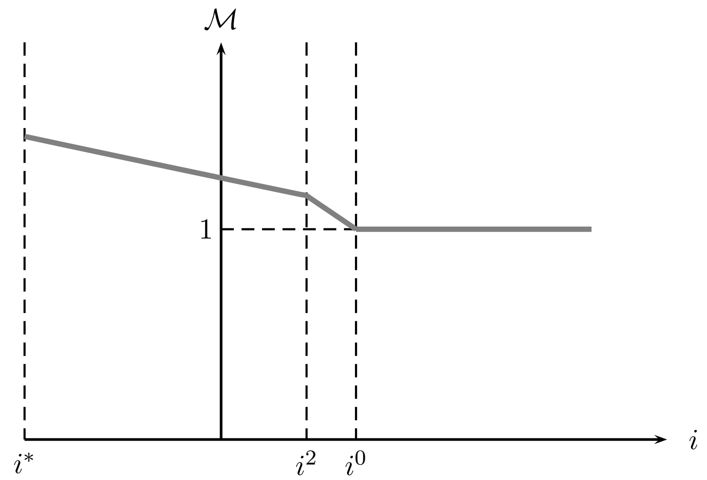

Figure 6: Money multiplier

6 Conclusion

We present a general equilibrium model of bank deposits and credit that combines two different features of banks. On the one hand, banks act as safe-keepers as in He, Huang, and Wright (2008), on the other hand banks act as intermediaries as in Berentsen, Camera, and Waller (2007). The interesting thing about the first aspect of banking is the fact that all agents are willing to deposit a measure of their wealth in a bank account. Hence, banks are already active in the centralized markets of the classic monetary theory model of Lagos and Wright (2005). Then, in a consecutive credit market, they act purely as transaction machines by using their unused deposits (from non-buyers) to issue bank loans to the cash constrained buyers. This allows them to increase their consumption and is therefore welfare improving.

The version with bank loans is presented together with a version without bank loans. The equilibria in the version without bank loans are different from the equilibria published in He, Huang, and Wright (2008) because we take into account the fees payed on a buyer's bank deposits. We find that the equilibrium interest rate boundaries are the same in the two versions. So, the lower boundary for a monetary equilibrium is equal to the negative of the transaction costs. The upper interest boundary, where there are no thieves and no banks anymore are equal as well. In between these boundaries, there is an interval of interest rates, where banks and theft co-exist. In contrast to the version without loans, the version with loans then has an equilibrium without theft but with banks.

This last equilibrium should be discussed and analyzed further in future versions of this paper. First of all, we see (especially in the equilibrium without theft but banks) that putting money on a bank account gives a potential for future credits as the bank may use unused deposits for issuing loans to agents. In other words, bank deposits may act as a kind of insurance against future preference shocks. As no agent knows ex ante whether he will become a buyer or a seller, every agent may have an interest to deposit some of his money as an insurance against becoming buyer. An other important point is that the bank might be able to finance a non-negative interest rate on deposits via charging higher interest rates from agents who demand loans. So, an interesting question to examine is, what happens if the bank does not charge a fee on bank deposits and neither pays an interest rate but uses the loan interest rate to finance all its expenses? We may already give a short answer by claiming that this case would probably lead to a non-monetary equilibrium with only bank deposits.

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